H-003-001513

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B. Sc. (Sem. V) (CBCS) Examination

May/June – 2017

Mathematics : BSMT-501(A)

(Mathematical Analysis - I & Group Theory)

Faculty Code : 003

Subject Code : 001513

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

1 Answer the following : **20**

(1) Define : Norm of a partition.

(2) Define : Refinement of a partition.

(3) Define : $U(P, f)$.

(4) State fundamental theorem of integration.

(5) State first mean value theorem of integral calculus.

(6) Define : Metric Space.

(7) Define : Accumulation Point.

(8) State Housedorff's property for metric spaces.

(9) Define : Countable Set.

(10) Define : Border Set.

(11) Define : Monoid.

(12) Define : Idempotent Element.

(13) Define : $SL(n; \mathbb{R})$.

(14) Write any two elementary properties of a group.

(15) State Euler's theorem.

(16) Define : Equivalence Relation.

(17) Define : Transposition.

(18) Define : Normal Subgroup.

(19) Define : Simple Group.

(20) Define : Inner Automorphism.

2 (a) Answer any three : **6**

(1) Define :

(i) Partition

(ii) Oscillatory Sum.

(2) State Darboux Theorem.

(3) $f(x) = \frac{20}{x}$, $x \in [2, 20]$, $p = \{2, 4, 5, 20\}$.

Find $U(P, f)$ and $L(P, f)$.

(4) Evaluate : $\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right) \ldots\ldots \left(1+\frac{4n}{n}\right)\right]^{\frac{1}{n}}$.

(5) If $E = \{x \in \mathbb{R} \ / \ 0 < x < 5\}$, then find E'.

(6) Give an example of subsets A and B of the metric space $\mathbb{R}$ such that $(A+B)^{\circ} \neq A^{\circ} + B^{\circ}$.

(b) Answer any three : **9**

(1) Prove that every continuous function is R-integrable.

(2) Prove that the sum of two R-integrable functions is R-integrable.

(3) Evaluate : $\frac{1}{n^2} \sum_{r=0}^{n-1} \sqrt{n^2 - r^2}$.

(4) State and prove the first mean value theorem for R-integration.

(5) Show that $\frac{3}{4}$ is in Cantor set.

(6) If A and B are subsets of a metric space (X, d), then prove that $\left(\overline{A+B}\right) = \overline{A+B}$.

(c) Answer any two : **10**

(1) State and prove the necessary and sufficient conditions for a function to be R-integrable.

(2) Evaluate $\int_0^1 f\, dx$ by showing that f is R-integrable in $[0, 1]$ where f is defined as follows :

$$f(x) = \frac{1}{a^r - 1} \;;\; \text{if } \frac{1}{a^r} < x \le \frac{1}{a^{r-1}},\; r = 1, 2, 3 \ldots\ldots$$
$$= 0 \quad ;\; \text{if } x = 0$$

where $a > 1$.

(3) Prove : $\lim_{n\to\infty} \left(\frac{n^n}{n!}\right)^{\frac{1}{n}} = e$.

(4) Prove that $(\mathbb{R}, d)$ is a separable metric space where d is the usual metric on $\mathbb{R}$.

(5) If (X, d) is a metric space, then prove that $\left(X, \frac{d}{1+d}\right)$ is also a metric space.

3 (a) Answer any three : **6**

(1) State and prove the reversal law in a group.

(2) Prove that a group G is commutative if $(ab)^2 = a^2b^2,\ \forall\ a, b \in G$.

(3) Show that a group of prime order cannot have a proper subgroup.

(4) Define : Symmetric Group.

(5) Show that $\left(\mathbb{R}^+, \bullet\right) \cong (\mathbb{R}, +)$.

(6) If $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 7 & 4 & 6 & 1 & 5 & 8 \end{pmatrix} \in S_8$, then find $o(\sigma)$.

(b) Answer any three : **9**

(1) Prove that every cyclic group is commutative. Also give an example to show that the converse does not hold.

(2) Define : Lattice Diagram. Draw the lattice diagram of $(\mathbb{Z}_6, +_6)$.

(3) Prove that a non-empty subset of H of a finite group G is a subgroup of G if the binary operation of G is closed in H.

(4) Prove that product of any two disjoint cycles in S_n is commutative.

(5) Let G be a group and let $a, b \in G$ such that $a \neq e$ and $o(b) = 2$. Find $o(a)$ if $bab^{-1} = a^2$.

(6) Show that a group cannot be a union of its two proper subgroups.

(c) Answer any two : **10**

(1) State and prove Lagrange's theorem. Let G be a group and $a \in G$.

(2) Prove that $H = \left\{ a^n \,/\, n \in \mathbb{Z} \right\}$ is the smallest subgroup of G containing a.

(3) State and prove Cayley's theorem.

(4) Define : Center of a group. If $\phi \in a(G)$ and Z is the center of G then prove that $\phi(Z) \subset Z$.

(5) If $f \in S_n$ is a cycle of odd order, then prove that f^2 is also a cycle.